Gravity Functions

https://lab.egeospace.in/gravity-functions/index.html

Familiar Version

Let $n,\ b,\ r \in \mathbb{Z}^+$ with $b \geq 2$ and $1 < r < b$

$$\text{ஞ}(n) = \begin{cases} \frac{n}{b}, & \text{if } n \equiv 0 \pmod{b} \\ (b+1)n+1, & \text{if } n \equiv 1 \pmod{b} \\ (n+1)b, & \text{if } n \equiv r \pmod{b} \end{cases}$$

$$a_{k+1} = \text{ஞ}(a_k), \quad a_0 = n$$

Efficient Version

Let $n,\ b,\ d,\ r \in \mathbb{Z}^+$ with $b \geq 2,\ 1 \leq d < b$ and $1 < r < b$

$$\text{ஞ}(n) = \begin{cases} \frac{n}{b}, & \text{if } n \equiv 0 \pmod{b} \\ (b-d)n+d, & \text{if } n \equiv 1 \pmod{b} \\ (b-d)n+d(n \mod b)-b, & \text{if } n \equiv r \pmod{b} \end{cases}$$

$$a_{k+1} = \text{ஞ}(a_k), \quad a_0 = n$$

Non-binary Version

Let $n,\ b,\ d,\ r \in \mathbb{Z}^{+}$ with $b > 2,\ 1 \leq d,\ d \neq b$ and $1 < r < b$

$$\text{ஞ}(n) = \begin{cases} \frac{n}{b}, & \text{if } n \equiv 0 \pmod{b} \\ (b+d)n - d, & \text{if } n \equiv 1 \pmod{b} \\ (n+1)b, & \text{if } n \equiv r \pmod{b} \end{cases}$$

$$a_{k+1} = \text{ஞ}(a_k), \quad a_0 = n$$

Notes

Observations of gravity functions

1. Deceptively projects itself as a base factor elimination algorithm, but is actually designed to convert every non-base factor into base factor. Efficient version highlights this: odd terms and even terms form a strictly decreasing sequence.
2. Because base factors are trivial, it's sufficient to understand non-base-multiple inputs.
3. Digit strings seem to play a critical role. Cases have to be designed to eliminate the remainder of an orbit.
4. Efficient version and non-binary version ensure ultimate cycle is $1 \rightarrow b \rightarrow 1$ while familiar version depends on base. Or simply, base and cases decide the cycle length post unity arrival.
5. Just like how factor partitions decide divisibility and introduce the concept of prime numbers, may be we are looking at a new property of numbers discovered and pointing at black holes. Unlike primes, this property is likely to be base-specific.
6. Cases have to be chosen to ensure convergence and avoid cycles. That is, function definition is not random; base dependent. That is, every natural number establishes a universal relationship with every prime!!!
7. Better starting point would be to consider digit strings, prime factors and partitions of input and their relationships with the chosen base.

Observations about gravity functions

1. Efficiency is high when base and multiplication delta are at their closest.
2. Though it seems base is the key, base proximity is also of value and needs to be studied.